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Effects of Green Technology Investment and Trade-Credit Facility in a Two-Warehouse Fuzzy Inventory Model Under Partially Backlogged Shortages

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Abstract: The role that green emissions play in the current ecosystem is extremely important because of the significant disruption that they cause to the environmental situation. Because of this, the majority of industrialised nations are making investments in programs that aim to facilitate environmentally friendly industrial processes. By taking into consideration the optimal investment in environmentally friendly technology to reduce the impact of carbon dioxide (CO2), which is primarily a contribution from the transportation of goods between warehouses and customers, this research develops two-warehouse (one is an own warehouse, and the other is a rented warehouse) crisp and fuzzy stockout models to manage stockout conditions with a single-level trade credit. To obtain the best total cost numerically, partial backlogging is taken into consideration in the model. This is because of the impacts of deterioration. In addition to this, an algorithmic approach to the process of finding a solution is offered. Once everything is said and done, a sensitivity analysis is carried out to assess the impact of modifications made to input parameters, providing valuable management insights.

Keywords: Two-Warehouse; Economic Order Quantity; Carbon Emission; One-Level Trade-Credit; Renting Warehouses; Owned Warehouses; Green Technology Investment; Intuitionistic Fuzzy; α, β -Cut Method.

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1. Introduction

Primarily, a study on the economic order quantity (EOQ) model was introduced by Harris [9]. The model was considered with minimal/basic assumptions, such as instantaneously receiving items with no shortages and a constant demand rate with unlimited storage capacity. However, a warehouse with unlimited storage capacity may be possible for local small businesses. Again, the demand for the items may change depending on time, price, stock, quality, season, etc. To manage fluctuations in demand and the global acceptance of businesses, managers often prefer renting warehouses (RW) and owning warehouses (OW). Some researchers, such as Nurhasril et al. [14] and Srabani and Chakrabarti [21], have developed an inventory model with two warehouse facilities, where the rate of demand is either time or selling-price-dependent. Management of environmentally efficient EOQ models has become a key interest for present-era strategic businesses, as environmental conditions are primarily influenced by the storage and transportation of items from warehouses to customers. Both holding and transportation contribute to carbon emissions (CE), drastically affecting the environment. To protect the environment from the damaging effects of carbon dioxide (CO2), researchers have taken an interest in providing sustainable EOQ models. Parida et al. [18], San-José et al. [12], Chaudhary et al. [16], Supakar et al. [15], Mashud et al. [4], and Yu et al. [8] have formulated inventory models to determine the sustainable inventory policy that optimises cost/profit. Again, in each of these inventory models, a carbon tax policy and a carbon cap-and-trade policy are implemented to encourage green inventories by implementing green technology investment (GTI), which reduces the effect of CO2 emissions.

Moreover, trade credit is common in today's competitive business scenario. Among various trade credits, a single-level trade credit (STC) enables retailers to receive credit periods from suppliers. Under STC, retailers instantly pay a percentage of the total purchasing cost (PC) after receiving the items. Then, the supplier allows a permissible delay in the payment period on the outstanding quantity. Panda et al. [10], San-José et al. [12], and Momena et al. [1] have studied inventory models with STC to obtain the optimum profit/cost. Due to the presence of uncertainty in decision-making situations, researchers adopt fuzzy concepts to obtain the required results. As an extension of the fuzzy set, the intuitionistic fuzzy set (IFS) concept has immense applications in decision-making, particularly in economic order/production quantity problems. IFS can be used in inventory models with uncertain demand patterns and uncertain factors influencing holding cost/purchasing cost/stockout cost, etc. In all these cases, uncertainties can be handled not only by the degree of membership but also by the degree of non-membership. In such situations, IFS helps to optimise the cost/profit while maintaining the service level. Sugapriya et al. [7], Parida et al. [19], Sahoo and Acharya [20], and Iqbal and Sarkar [13] included an IF environment in economic order/production quantity (EPQ) models to achieve the optimum cost/profit. For a two-warehouse deteriorating model with shortage, we intend to know:

- The optimum green technology (GT) cost
- The optimum time-cycle length
- The total cost with the single trade credit option for the retailer
- A set of managerial insights

Keeping in view these, the main novelty of the work lies in its complete derivation of a two-warehouse EOQ model under the optimum GT investments with the trade credit option for retailers. This research studies a deteriorating EOQ model with partially backlogged shortages and a permissible late payment option with interest earned (IE) from the revenue after fulfilling the shortages at the initial point of the cycle and interest paid by the retailer on the unpaid balance. The proposed model optimises the retailer's replenishment cycle and GT cost, obtaining the total average cost, where the optimality criteria are verified by taking an appropriate numerical example. Again, the effects of specific parameters on the optimum total cost are observed for the decision-making situations (Table 1).

Table 1: Notations and nomenclature used in the mathematical model

Nomenclature			
b_q	Backorder units	m_{tc_3}	Extra fuel consumed per ton of load
Q_{oq}	Number of items ordered	p_w	Product weight
n_i	Number of payments	T_n	Trip covered
m_{tc_1}	The least transportation cost	$\hat{a}$	Initial rate of demand
m_{tc_2}	Fuel used for an empty vehicle	$\hat{b}$	The rate at which the demand rate increases
$\hat{c}$	The rate at which the change in the demand rate itself decreases	C_{ow}	Capacity of OW
L_t	Lead time	E_i	Interest earned (IE)
t_0	Deterioration-free period	t_2	Shortage-free period
$\hat{D}$	Distance travelled	V_t'	Fuel price
c_{e1}	Cost of the emissions from the vehicle	c_{e2}	Cost for emission by transferring one unit of an item
$\hat{C}_O$	Per unit ordering cost	$\hat{p}_c$	Per unit purchasing cost
H_{cw}	Carrying cost for OW	H_{cr}	Carrying cost for RW
P_{sc}	Shortage cost/ unit	P_{dc}	Disposal cost/ unit
C_c	Capital cost	R_{tc}	Decreased transportation cost
s_p	Selling price/ unit	M_R	Credit period for the retailer
r_{ip}	Interest paid	α	Rate of shortages
θ	The deterioration rate in the case of RW	β	Deterioration rate in OW
Decision Variables (DV)			
G_{tc}	GT investment cost		
T_{sl}	Total cycle length		

T_{oc} / $\widetilde{T}_{oc}$	Total inventory cost in both environments

2. Definitions

Intuitionistic Fuzzy Number (IFN) [13]: "Let X be a non-empty set of discourses. $\widetilde{B}$ be an intuitionistic fuzzy set in X given by ordered triples of an object x, where the degree of belongingness is $\mu_{\widetilde{B}}(x)$ and the degree of non-belongingness is $v_{\widetilde{B}}(x)$ of $\widetilde{B}$. An intuitionistic fuzzy number is defined by $\widetilde{B} = \{< x, \mu_{\widetilde{B}}(x), v_{\widetilde{B}}(x) >: x \in X\}$, where $\mu_{\widetilde{B}}(x), v_{\widetilde{B}}(x): X \to [0,1]$ for every $x \in X_{\widetilde{B}}$ with $0 \le \mu_{\widetilde{B}}(x) + v_{\widetilde{B}}(x) \le 1$. The value of $\pi_{\widetilde{B}}(x) = 1 - \mu_{\widetilde{B}}(x) - v_{\widetilde{B}}(x)$ is called the IF index of $x \in U$ for $\widetilde{B}$."

Triangular Intuitionistic Fuzzy Number (TIFN) [5]: "TIFNs are a special type of IFN. Let $\widetilde{B}_t$ be a triangular Intuitionistic fuzzy number defined by $\widetilde{B}_t = (b_1, b_2, b_3; b_1', b_2, b_3')$, where $b_1' \le b_1 \le b_2 \le b_3 \le b_3'$ on $\mathfrak{R}$. The following are the membership ($\mu_{\widetilde{B}_t}(x)$) and non-membership functions, respectively, of the IF number $\widetilde{B}_t$".

$$\mu_{\widetilde{B}_t}(x) = \begin{cases} \frac{x-b_1}{b_2-b_1}, b_1 \le x \le b_2 \\ \frac{b_3-x}{b_3-b_2}, b_2 \le x \le b_3, \\ 0, \text{otherwise} \end{cases} \qquad v_{\widetilde{B}_t}(x) = \begin{cases} \frac{b_2-x}{b_2-b_1'}, b_1' \le x \le b_2 \\ \frac{x-b_2}{b_3'-b_2}, b_2 \le x \le b_3' \\ 1, \text{otherwise} \end{cases}$$

Defuzzification Method for TIFN by αβ-cut [5]: "Let $\widetilde{B}_t = (b_1, b_2, b_3; b_1', b_2, b_3')$, where $b_1' \le b_1 \le b_2 \le b_3 \le b_3'$ on $\mathbb{R}$. The α-cut of the membership function $\mu_{\widetilde{B}_t}(x)$, for $\alpha \in [0,1]$. $B(\alpha) = [b_1 + \alpha(b_2 - b_1), b_3 - \alpha(b_3 - b_2)] = [B_L(\alpha), B_R(\alpha)]$ and β −cut of the non-membership function $v_{\widetilde{B}_t}(x)$ for $\beta \in [0,1]$, we get $B(\beta) = [b_2 - \beta(b_2 - b_1'), b_2 + \beta(b_3' - b_2)] = [B_L(\beta), B_R(\beta)]$. Defuzzifying the membership function $\mu_{\widetilde{B}_t}(x)$ and the non-membership function $v_{\widetilde{B}_t}(x)$, known as the score functions (Table 2), we get:

$$D_f(\mu_{\widetilde{B}_t}(x)) = \int_0^1 \frac{B_L(\alpha)+B_R(\alpha)}{2} d\alpha = \frac{2b_2+(b_1+b_3)}{4}, D_f(v_{\widetilde{B}_t}(x)) = \int_0^1 \frac{B_L(\beta)+B_R(\beta)}{2} d\beta = \frac{2b_2+(b_1'+b_3')}{4}$$

Hence, equation (1) represents the defuzzification formula for TIFNs."

$$D_f(\widetilde{B}_t) = \frac{1}{2}\left[D_f\left(\mu_{\widetilde{B}_t}(x)\right) + D_f\left(v_{\widetilde{B}_t}(x)\right)\right] = \frac{1}{2}\left[b_2 + \frac{1}{4}(b_1 + b_1' + b_3 + b_3')\right] \quad (1)$$

Table 2: Comparison table between the existing and present research

References	**Two warehouses**	**Time-dependent demand**	**Partial back-logging**	**GTI towards carbon emission**	**Transportation is responsible for emissions**	**Cost for disposal**	**Intuitionistic fuzzy**	**With STC**
Ghosh et al. [2]	Y	-	Y	-	-	-	-	-
Guria et al. [3]	Y	Y	-	-	-	-	-	-
Tiwari et al. [22]	Y	-	Y	-	-	-	-	Y
Chung [11]	Y	-	Y	Y	Y	-	-	-
Chaudhary et al. [16]	-	-	Y	-	-	-	Y	-
Singh and Yadav [17]	-	-	Y	Y	-	-	Y	-
This Work	Y	Y	Y	Y	Y	Y	Y	Y

3. Mathematical Formulations

The following are the assumptions for the models:

- **Partial backlogging rate:** α
- **Rate of demand for customers:** $D = \begin{cases} (\hat{a} + \hat{b}t - \hat{c}t^2), Q(t) \ge 0 \\ \alpha(\hat{a} + \hat{b}t - \hat{c}t^2), Q(t) < 0 \end{cases}$

- The supplier gives the retailer a credit period. M_R, where$0 \leq M_R \leq T_{sl}$. The following are the two different situations that could occur:

  - The trade-credit time is within the stock period, i.e., $[0, t_2]$.
  - The trade-credit time is outside the stock period, i.e., $[t_2, T_{sl}]$.

- For the present problem, the account is settled within the period $[0, T_{sl}]$ in which the retailer earns interest at the rate of $E_i$within the stock period $[0, t_2]$ which is the trade credit period offered by the supplier, and when the time is beyond t_2, the retailer starts paying the interest at the rate of r_{ip} to the supplier.

At time t, $Q_{rw}(t)$ and $Q_{ow}(t)$ in the RW and OW are obtained by solving the Eqs. (2) and (4) -(6) concerning the conditions mentioned in (3) and (7) respectively:

For RW: $\frac{dQ_{rw}}{dt} + \theta Q_{rw} = -(\hat{a} + \hat{b}t - \hat{c}t^2), 0 \leq t \leq t_0$ (2)

Where the boundary conditions are, $Q_{rw}(0) = C_R - C_{ow}$ and $Q_{rw}(t_0) = 0$ (3)

For OW: $\frac{dQ_{ow}}{dt} + \beta Q_{ow} = 0, 0 \leq t \leq t_0$ (4)

$$\frac{dQ_{ow}}{dt} + \beta Q_{ow} = -(\hat{a} + \hat{b}t - \hat{c}t^2), t_0 \leq t \leq t_2 \quad (5)$$

$$\frac{dQ_{ow}}{dt} = -(\hat{a} + \hat{b}t - \hat{c}t^2)\alpha, t_2 \leq t \leq T_{sl} \quad (6)$$

With $Q_{ow}(0) = C_{ow}, Q_{ow}(t_2) = 0, Q_{ow}(T_{sl}) = -b_q$ (7)

The solutions of Eqs. (2), and (4) -(6) concerning the boundary conditions (3) and (7) are given in Eqs. (8) and (9) -(11) respectively.

$$Q_{rw}(t) = -\frac{\hat{a}}{\theta} + \frac{\hat{b}}{\theta^2} - \frac{\hat{b}t}{\theta} + \frac{\hat{c}t^2}{\theta} - \frac{2\hat{c}t}{\theta^2} + \frac{2\hat{c}}{\theta^3} + (S_R - S_O + \frac{\hat{a}}{\theta} - \frac{\hat{b}}{\theta^2} - \frac{2\hat{c}}{\theta^3})(1 - \theta t), 0 \leq t \leq t_0 \quad (8)$$

$$Q_{ow}(t) = S_O e^{-\beta t}, 0 \leq t \leq t_0 \quad (9)$$

$$Q_{ow}(t) = -\frac{\hat{b}t}{\beta} + \frac{\hat{c}t^2}{\beta} - \frac{2\hat{c}t}{\beta^2} + \frac{\hat{b}t_2}{\beta} - \frac{\hat{c}t_2^2}{\beta} + \frac{2\hat{c}t_2}{\beta^2}, t_0 \leq t \leq t_2 \quad (10)$$

$$Q_{ow}(t) = \alpha(\hat{a}t + \frac{\hat{b}t^2}{2} - \frac{\hat{c}t^3}{3} + \hat{a}T_{sl} + \frac{\hat{b}{T_{sl}}^2}{2} - \frac{\hat{c}{T_{sl}}^3}{3}) - b_q, t_2 \leq t \leq T_{sl} \quad (11)$$

Again, using equation (3) for $t = 0$ in Eq. (8) gives:

$$C_R = C_O + \frac{1}{1-\theta t_0}\left(\hat{a}t_0 - \frac{\hat{c}t_0^2}{\theta}\right) \quad (12)$$

Now, by the continuity condition at points $t = t_0$and $t = t_2$, we get:

$$C_R e^{-\beta t_0} = -\frac{bt_0}{\beta} + \frac{\hat{c}t_0^2}{\beta} - \frac{2\hat{c}t_0}{\beta^2} + \frac{bt_2}{\beta} - \frac{\hat{c}t_2^2}{\beta} + \frac{2\hat{c}t_2}{\beta} \quad (13)$$

And $b_q = \alpha\left(\hat{a}T_{sl} + \frac{\hat{b}T_{sl}}{2} - \frac{\hat{c}T_{sl}}{3} - \hat{a}t_2 - \frac{\hat{b}t_2^2}{2} + \frac{\hat{c}t_2^3}{3}\right)$ (14)

So, the per-cycle order quantity is:

$$Q_{oq} = S_R + b_q = S_O + \frac{1}{1-\theta t_0}\left(\hat{a}t_0 - \frac{\hat{c}t_1^2}{\theta}\right) + \alpha\left(\pi\hat{a}T_{sl} + \frac{\hat{b}T_{sl}}{2} - \frac{\hat{c}T_{sl}}{3} - \hat{a}t_2 - \frac{\hat{b}t_2^2}{2} + \frac{\hat{c}t_2^3}{3}\right) \quad (15)$$

The following are the inventory-related costs:

- **Cost due to ordering:** $\widehat{C}_O$
- **Total cost towards purchasing:** $T_{pc} = \hat{p}_c Q_{oq} = \hat{p}_c(C_R + b_q)$ (16)
- **Total carrying costs for RW and OW:** $T_{HC} = H_{cr}\int_0^{t_0} Q_{rw}(t)dt + H_{cw}\int_0^{t_2} Q_{ow}(t)dt$

$$= H_{cr}[\frac{\hat{c}t_0^3}{3\theta} + (C_R - C_{ow})(t_0 - \frac{\theta t_0^2}{2}) - \frac{\hat{a}t_0^2}{2}] + H_{cw}[C_R t_0 + \frac{\hat{b}(t_2-t_0)^2}{2\beta} + \frac{\hat{c}}{\beta}\{\frac{(t_2-t_0)^2}{\beta} - \frac{2t_2^3}{3} - \frac{t_0^3}{3} + t_2^2 t_0\}] \quad (17)$$

- **Total cost related to the shortage:** $T_{sc} = -P_{sc}\int_{t_2}^{T_{sl}} Q_{ow}(t)dt = P_{sc}\alpha[-\frac{3\hat{a}(T_{sl}-t_2)^2}{2} - \frac{5\hat{b}T_{sl}^3}{6} - \frac{2\hat{b}t_2^3}{3} + \frac{7\hat{c}T_{sl}^4}{12} + \frac{5\hat{c}t_2^4}{12} + \hat{b}T_{sl}^2 t_2 + \frac{\hat{b}T_{sl}t_2^2}{2} - \frac{\hat{c}t_2^3 T_{sl}}{3} - \frac{2\hat{c}T_{sl}^3 t_2}{3}]$ (18)
- **Cyclic capital cost:** $P_{cc} = (\frac{n_i+1}{2n_i})P_i \mu L_t \hat{p}_c Q_{oq}$ (19)
- **Total cost due to transportation (includes both fixed and variable transportation charges and CE costs):**

$$T_{nc} = \frac{T_n}{T_{sl}}[m_{tc_1} + (2\widehat{D}V_t' m_{tc_2} + \widehat{D}\widehat{m}_{tc_3}V_t' p_w Q_{oq}) + (2\widehat{D}c_{e1} + \widehat{D}c_{e2}Q_{oq})] \quad (20)$$

- **GTI cos [4]:** $A_{gc} = G_{tc}T_{sl}$ (21)
- **Green logistics cost:**

$$D_{TC} = \frac{T_n}{T_{sl}}[m_{tc_1} + (2\widehat{D}V_t' m_{tc_2} + \widehat{D}\widehat{m}_{tc_3}V_t' p_w Q_{oq}) + ((2\widehat{D}c_{e1} + \widehat{D}c_{e2}Q_{oq})(1 - \xi'(1 - e^{-\chi' G_{tc}})))] \quad (22)$$

- **Disposal cost [9]:**

$$T_{DC} = P_{dc}n_d = P_{dc}[\theta\int_0^{t_0} Q_{rw}(t)dt + \beta\int_0^{t_0} Q_{ow}(t)dt + \beta\int_{t_0}^{t_2} Q_{ow}(t)dt] = P_{dc}[-2\hat{a}t_0 + \frac{\hat{a}t_1^2\theta}{2} - S_R e^{-\beta t_0}$$
$$-(t_2 - t_0)(\frac{\hat{b}}{2} + \hat{b}t_2 - \hat{c}t_2^2) + \hat{c}(\frac{t_2^3}{3} - \frac{(t_2^2-t_1^2)}{\beta} + \frac{2t_2(t_2-t_1)}{\beta}) + (S_R - S_O)(\theta t_0 - \frac{\theta^2 t_0^2}{2})] \quad (23)$$

Now, the supplier extends a permissible delay in payments to the retailer; thus, there may be two cases:

- $0 < M_R \le t_0 < t_2 < T_{sl}$
- $0 \le t_0 < M_R \le t_2 < T_{sl}$

Now, the detailed discussions of the above cases are given below. IP and IE by the retailer [6].

Case-1: IE from the revenue after covering the unsatisfying demand at the initial point of each cycle and IE from sales from 0 to M_R:

$$I_{E1} = s_p E_i[[\int_0^{M_R} D(t)dt] \times \int_{t_2}^{T_{sl}} dt + \int_0^{M_R}\int_0^t(\hat{a} + \hat{b}u - \hat{c}u^2)dudt]$$
$$= s_p E_i[[\hat{a}M_R + \frac{\hat{b}M_R^2}{2} - \frac{\hat{c}M_R^3}{3}](T_{sl} - t_2) + [\frac{\hat{a}M_R^2}{2} + \frac{\hat{b}M_R^3}{6} - \frac{\hat{c}M_R^2}{12}]]$$

.

IP in the unused inventory in RW from M_R to t_0:

$$I_{p1} = r_{ip}\hat{p}_c\left(\int_{M_R}^{t_0} Q_{rw}(t)dt\right) = r_{ip}\hat{p}_c\left[\frac{\hat{c}(t_0^3 - M_R^3)}{3\theta} + (C_R - C_{ow})\left(t_0 - M_R - \frac{\theta t_0^2}{2} + \frac{\theta M_R^2}{2}\right) - \frac{\hat{a}(t_0^2 - M_R^2)}{2}\right].$$

The retailer also clears the dues for the stock stored in OW from the period M_R to t_0 and after that, interest is paid for the remaining inventory in the period t_0 to t_2 are:

$$I_{p2} = \hat{p}_c r_{ip}\left[C_{ow}\int_{M_R}^{t_0} dt + \int_{t_0}^{t_2} Q_{ow}(t)dt\right]$$

$$= \hat{p}_c r_{ip}[C_{ow}(t_0 - M_R) + (t_2^2 - t_0^2)\left(-\frac{\hat{b}}{2\beta} - \frac{\hat{c}}{\beta^2}\right) + \frac{\hat{c}}{3\beta}(t_2^3 - t_0^3) + (t_2 - t_0)\left(\frac{\hat{b}t_2}{\beta} - \frac{\hat{c}t_2^2}{\beta} + \frac{2\hat{c}t_2}{\beta^2}\right)]$$

Case-2: The total IE on the revenue after fulfilling the shortages at the beginning of the cycle and from 0 to M_R in OW:

$I_{E1} = s_p E_i \int_0^{M_R} D(t)dt \times \int_{t_2}^{T_{sl}} dt + \int_0^{M_R}\int_0^t D(u)dudt = s_p E_i[\hat{a}M_R + \frac{\hat{b}M_R^2}{2} - \frac{\hat{c}M_R^3}{3}](T_{sl} - t_2) + [\frac{\hat{a}M_R^2}{2} + \frac{\hat{b}M_R^3}{6} - \frac{\hat{c}M_R^2}{12}]$.
And IP for the unsold units stored in OW during M_R to t_2 is:

$$I_{p1} = r_{ip}\hat{p}_c\left(\int_{M_R}^{t_2} Q_{ow}(t)dt\right) = r_{ip}\hat{p}_c\left[(t_2^2 - M_R^2)\left(-\frac{\hat{b}}{2\beta} - \frac{\hat{c}}{\beta^2}\right) + \frac{\hat{c}}{3\beta}(t_2^3 - M_R^3) + (t_2 - M_R)\left(\frac{\hat{b}t_2}{\beta} - \frac{\hat{c}t_2^2}{\beta} + \frac{2\hat{c}t_2}{\beta^2}\right)\right]$$

We use the above appropriate costs to obtain T_{oc} for both case 1 and case 2 for the crisp model:

$$T_{oc} = \frac{1}{T_{sl}}[\hat{C}_O + T_{pc} + T_{HC} + T_{sc} + P_{cc} + T_{nc} + A_{gc} + D_{TC} + T_{DC} + I_{p1} + I_{p2} - I_{E1}] \tag{24}$$

Note: Using a similar total cost formula, as mentioned in Eq. (24), we obtain the IF model, which incorporates IF parameters instead of crisp parameters at the respective costs.

Case-1

$$T_{oc} = \frac{1}{T_{sl}}[\hat{C}_O + \hat{p}_c\left[C_{ow} + \frac{1}{1-\theta t_0}\left(\hat{a}t_0 - \frac{\hat{c}t_1^2}{\theta}\right) + \alpha\left(\hat{a}T_{sl} + \frac{\hat{b}T_{sl}}{2} - \frac{\hat{c}T_{sl}}{3} - \hat{a}t_2 - \frac{\hat{b}t_2^2}{2} + \frac{\hat{c}t_2^3}{3}\right)\right] + H_{cr}[\frac{\hat{c}t_0^3}{3\theta} + (C_R - C_{ow})(t_0 - \frac{\theta t_0^2}{2}) - \frac{\hat{a}t_0^2}{2}] + H_{cw}S_R t_0 + H_{cw}[\frac{\hat{b}t_2^2}{2\beta} + \frac{\hat{b}t_0^2}{2\beta} - \frac{2\hat{c}t_2^3}{3\beta} - \frac{\hat{c}t_0^3}{3\beta} - \frac{\hat{c}t_2^2}{\beta^2} + \frac{\hat{c}t_0^2}{\beta^2} - \frac{\hat{b}t_2 t_0}{\beta} + \frac{\hat{c}t_2^2 t_0}{\beta} + \frac{2\hat{c}t_2^2}{\beta^2} - \frac{2\hat{c}t_2 t_0}{\beta^2}] + P_{sc}\alpha[-\frac{3\hat{a}T_{sl}^2}{2} - \frac{3\hat{a}t_2^2}{2} - \frac{5\hat{b}T_{sl}^3}{6} - \frac{2\hat{b}t_2^3}{3} + \frac{7\hat{c}T_{sl}^4}{12} + \frac{5\hat{c}t_2^4}{12} + 3\hat{a}T_{sl}t_2 + \hat{b}T_{sl}^2 t_2 + \frac{\hat{b}T_{sl}t_2^2}{2} - \frac{\hat{c}t_2^3 T_{sl}}{3} - \frac{2\hat{c}T_{sl}^3 t_2}{3}] + (\frac{n_i+1}{2n_i})P_i\mu L_t\hat{p}_c Q_{oq} + \frac{T_n}{T_{sl}}[m_{tc_1} + (2\hat{D}V_t' m_{tc_2} + \hat{D}\hat{m}_{tc_3}V_t' p_w Q_{oq})(1 - \xi'(1 - e^{-\chi'\hat{G}_c})))] + P_{dc}[[-2\hat{a}t_0 + \frac{\hat{a}t_1^2\theta}{2} - C_R e^{-\beta t_0} - \frac{\hat{b}(t_2 - t_0)}{2} + \frac{\hat{c}t_2^3}{3} - \frac{\hat{c}(t_2^2 - t_1^2)}{\beta} + \hat{b}t_2(t_2 - t_0) - \hat{c}t_2^2(t_2 - t_0) + \frac{2\hat{c}t_2(t_2 - t_1)}{\beta} + (C_R - C_{ow})(\theta t_0 - \frac{\theta^2 t_0^2}{2})] - s_p E_i[\hat{a}M_R + \frac{\hat{b}M_R^2}{2} - \frac{\hat{c}M_R^2}{12}] + r_{ip}\hat{p}_c[\frac{\hat{c}(t_0^3 - M_R^3)}{3\theta} + (C_R - C_{ow})\left(t_0 - M_R - \frac{\theta t_0^2}{2} + \frac{\theta M_R^2}{2}\right) - \frac{\hat{a}(t_0^2 - M_R^2)}{2}] + \hat{p}_c r_{ip}C_{ow}(t_0 - M_R) + r_{ip}\hat{p}_c[(t_2^2 - t_0^2)(-\frac{\hat{b}}{2\beta} - \frac{\hat{c}}{\beta^2}) + \frac{\hat{c}}{3\beta}(t_2^3 - t_0^3) + (t_2 - t_0)\left(\frac{\hat{b}t_2}{\beta} - \frac{\hat{c}t_2^2}{\beta} + \frac{2\hat{c}t_2}{\beta^2}\right)] + G_{tc}T_{sl}]$$

(25)

Case-2

$$T_{oc} = \frac{1}{T_{sl}}[\hat{C}_O + \hat{p}_c\left[C_{ow} + \frac{1}{1-\theta t_0}\left(\hat{a}t_0 - \frac{\hat{c}t_1^2}{\theta}\right) + \alpha(\hat{a}T_{sl} + \frac{\hat{b}T_{sl}}{2} - \frac{\hat{c}T_{sl}}{3} - \hat{a}t_2 - \frac{\hat{b}t_2^2}{2} + \frac{\hat{c}t_2^3}{3})\right] + H_{cr}[\frac{\hat{c}t_0^3}{3\theta} + (C_R - C_{ow})\quad(t_0 - \frac{\theta t_0^2}{2}) - \frac{\hat{a}t_0^2}{2}] + H_{cw}S_R t_0 + H_{cw}[\frac{\hat{b}t_2^2}{2\beta} + \frac{\hat{b}t_0^2}{2\beta} - \frac{2\hat{c}t_2^3}{3\beta} - \frac{\hat{c}t_0^3}{3\beta} - \frac{\hat{c}t_2^2}{\beta^2} + \frac{\hat{c}t_0^2}{\beta^2} - \frac{\hat{b}t_2 t_0}{\beta} + \frac{\hat{c}t_2^2 t_0}{\beta} + \frac{2\hat{c}t_2^2}{\beta^2} - \frac{2\hat{c}t_2 t_0}{\beta^2}] + P_{sc}\alpha[-\frac{3\hat{a}T_{sl}^2}{2} - \frac{3\hat{a}t_2^2}{2} - \frac{5\hat{b}T_{sl}^3}{6} - \frac{2\hat{b}t_2^3}{3} + \frac{7\hat{c}T_{sl}^4}{12} + \frac{5\hat{c}t_2^4}{12} + 3\hat{a}T_{sl}t_2 + \hat{b}T_{sl}^2 t_2 + \frac{\hat{b}T_{sl}t_2^2}{2} - \frac{\hat{c}t_2^3 T_{sl}}{3} - \frac{2\hat{c}T_{sl}^3 t_2}{3}] + (\frac{n_i+1}{2n_i})P_i\mu L_t\hat{p}_c Q_{oq} + \frac{T_n}{T_{sl}}[m_{tc_1} + (2\hat{D}V_t' m_{tc_2} + \hat{D}\hat{m}_{tc_3}V_t' p_w Q_{oq})(1 - \xi'(1 - e^{-\chi'\hat{G}_c})))] + P_{dc}[[-2\hat{a}t_0 + \frac{\hat{a}t_1^2\theta}{2} - C_R e^{-\beta t_0} - \frac{\hat{b}(t_2 - t_0)}{2} + \frac{\hat{c}t_2^3}{3} + \hat{b}t_2(t_2 - t_0) - \hat{c}t_2^2(t_2 - t_0) + \frac{2\hat{c}t_2(t_2 - t_1)}{\beta} + (C_R - C_{ow})(\theta t_0 - \frac{\theta^2 t_0^2}{2})] - s_p E_i[\hat{a}M_R + \frac{\hat{b}M_R^2}{2} - \frac{\hat{c}M_R^3}{3})(T_{sl} - t_2) + \frac{\hat{a}M_R^2}{2} + \frac{\hat{b}M_R^3}{6} - \frac{\hat{c}M_R^2}{12}] + r_{ip}\hat{p}_c[(t_2^2 - M_R^2)\left(-\frac{\hat{b}}{2\beta} - \frac{\hat{c}}{\beta^2}\right) + \frac{\hat{c}}{3\beta}(t_2^3 - M_R^3) + (t_2 - M_R)\left(\frac{\hat{b}t_2}{\beta} - \frac{\hat{c}t_2^2}{\beta} + \frac{2\hat{c}t_2}{\beta^2}\right)] \tag{26}$$

IF Model: Here, we consider $\hat{p}_c$, C_o and P_{sc} to be TIFN and are given as follows.

$\tilde{\hat{p}}_c = (\hat{p}_{c1}, \hat{p}_{c2}, \hat{p}_{c3}; \hat{p}_{c1}^1, \hat{p}_{c2}, \hat{p}_{c3}')$, $\tilde{\hat{C}}_o = (\hat{C}_{o1}, \hat{C}_{o2}, \hat{C}_{o3}; \hat{C}_{o1}^1, \hat{C}_{o2}, \hat{C}_{o3}^1)$ and $P_{sc} = (\tilde{P}_{sc1}, \tilde{P}_{sc2}, \tilde{P}_{sc3}; \tilde{P}_{sc1}^1, \tilde{P}_{sc2}, \tilde{P}_{sc3}^1)$.

Hence, $\tilde{T}_{oc}$ in IF sense is:

Case-1

$$\tilde{T}_{oc} = \frac{1}{T_{sl}}[\frac{1}{2}\{\hat{C}_{o2} + \frac{1}{4}(\hat{C}_{o1} + \hat{C}_{o1}' + \hat{C}_{o3} + \hat{C}_{o3}')\} + \frac{1}{2}\{\hat{p}_{c2} + \frac{1}{4}(\hat{p}_{c1} + \hat{p}_{c1}' + \hat{p}_{c3} + \hat{p}_3')\}[C_{ow} + \frac{1}{1-\theta t_0}(\hat{a}t_0 - \frac{\hat{c}t_1^2}{\theta}) +$$

$$\frac{\hat{b}T_{sl}}{2}-\frac{\hat{c}T_{sl}}{3}-\hat{a}t_2-\frac{\hat{b}t_2^2}{2}+\frac{\hat{c}t_2^3}{3})]+H_{cr}[\frac{\hat{c}t_0^3}{3\theta}+(C_R-C_{ow})(t_0-\frac{\theta t_0^2}{2})-\frac{\hat{a}t_0^2}{2}]+H_{cw}C_Rt_0+H_{cw}[\frac{\hat{b}t_2^2}{2\beta}+\frac{\hat{b}t_0^2}{2\beta}-\frac{2\hat{c}t_2^3}{3\beta}$$
$$\frac{\hat{c}t_0^2}{\beta^2}-\frac{\hat{b}t_2t_0}{\beta}+\frac{\hat{c}t_2^2t_0}{\beta}+\frac{2\hat{c}t_2^2}{\beta^2}-\frac{2\hat{c}t_2t_0}{\beta^2}]+\frac{1}{2}\{P_{sc2}+\frac{1}{4}(P_{sc1}+P'_{sc1}+P_{sc3}+P'_{sc3})\}\alpha[-\frac{3\hat{a}T_{sl}^2}{2}-\frac{3\hat{a}t_2^2}{2}-\frac{5\hat{b}T_{sl}^3}{6}-\frac{2\hat{b}t_2^3}{3}\quad\frac{5\hat{c}t_2^4}{12}+3\hat{a}T_{sl}t_2+$$
$$\hat{b}T_{sl}^2t_2+\frac{\hat{b}T_{sl}t_2^2}{2}-\frac{\hat{c}t_2^3T_{sl}}{3}-\frac{2\hat{c}T_{sl}^3t_2}{3}]+(\frac{n_i+1}{2n_i})P_i\mu L_t\frac{1}{2}\{\hat{p}_{c2}+\frac{1}{4}(\hat{p}_{c1}+\hat{p}'_{c1}+\hat{p}_{c3}+\hat{p}'_3)\}Q_{oq}+\frac{T_n}{T_{sl}}[m_{tc_1}+(2\hat{D}V'_tm_{tc_2}+$$
$$\hat{D}\hat{m}_{tc_3}V'_tp_wQ_{oq})+((2\hat{D}c_{e1}+\hat{D}c_{e2}Q_{oq})(1-\xi'(1-e^{-\chi'\hat{G}_c}))-\frac{\hat{b}(t_2-t_0)}{2})]+G_{tc}T_{sl}+P_{dc}[-2\hat{a}t_0+\quad\frac{\hat{a}t_1^2\theta}{2}-C_Re^{-\beta t_0}+\frac{\hat{c}t_2^3}{3}-$$
$$\frac{\hat{c}(t_2^2-t_1^2)}{\beta}+\hat{b}t_2(t_2-t_0)-\hat{c}t_2^2(t_2-t_0)+\frac{2\hat{c}t_2(t_2-t_1)}{\beta}+(C_R-C_{ow})(\theta t_0-\frac{\theta^2t_0^2}{2})]-s_pE_i[(\hat{a}M_R+\frac{\hat{b}M_R^2}{2}-\frac{\hat{c}M_R^3}{3})(T_{sl}-t_2)+\frac{\hat{a}M_R^2}{2}+$$
$$\frac{\hat{b}M_R^3}{6}-\frac{\hat{c}M_R^2}{12}]+r_{ip}\frac{1}{2}\{\hat{p}_{c2}+\frac{1}{4}(\hat{p}_{c1}+\hat{p}'_{c1}+\hat{p}_{c3}+\hat{p}'_3)\}[\frac{\hat{c}(t_0^3-M_R^3)}{3\theta}+(C_R-C_{ow})\left(t_0-M_R-\frac{\theta t_0^2}{2}+\frac{\theta M_R^2}{2}\right)-\frac{\hat{a}(t_0^2-M_R^2)}{2}+C_{ow}(t_0-$$
$$M_R)+(t_2^2-t_0^2)(-\frac{\hat{b}}{2\beta}-\frac{\hat{c}}{\beta^2})+\frac{\hat{c}}{3\beta}(t_2^3-t_0^3)\quad+(t_2-t_0)\left(\frac{\hat{b}t_2}{\beta}-\frac{\hat{c}t_2^2}{\beta}+\frac{2\hat{c}t_2}{\beta^2}\right)]] \quad (27)$$

Case-2

$$\tilde{T}_{oc}=\frac{1}{T_{sl}}[\frac{1}{2}\{\hat{C}_{o2}+\frac{1}{4}(\hat{C}_{o1}+\hat{C}'_{o1}+\hat{C}_{o3}+\hat{C}'_{o3})\}+\frac{1}{2}\{\hat{p}_{c2}+\frac{1}{4}(\hat{p}_{c1}+\hat{p}'_{c1}+\hat{p}_{c3}+\hat{p}'_3)\}[C_{ow}+\frac{1}{1-\theta t_0}(\hat{a}t_0-\frac{\hat{c}t_1^2}{\theta})+$$
$$\frac{\hat{b}T_{sl}}{2}-\frac{\hat{c}T_{sl}}{3}-\hat{a}t_2-\frac{\hat{b}t_2^2}{2}+\frac{\hat{c}t_2^3}{3})]+H_{cr}[\frac{\hat{c}t_0^3}{3\theta}+(C_R-C_{ow})(t_0-\frac{\theta t_0^2}{2})-\frac{\hat{a}t_0^2}{2}]+H_{cw}C_Rt_0+H_{cw}[\frac{\hat{b}t_2^2}{2\beta}+\frac{\hat{b}t_0^2}{2\beta}-\frac{2\hat{c}t_2^3}{3\beta}$$
$$\frac{\hat{c}t_0^2}{\beta^2}-\frac{\hat{b}t_2t_0}{\beta}+\frac{\hat{c}t_2^2t_0}{\beta}+\frac{2\hat{c}t_2^2}{\beta^2}-\frac{2\hat{c}t_2t_0}{\beta^2}]+\frac{1}{2}\{P_{sc2}+\frac{1}{4}(P_{sc1}+P'_{sc1}+P_{sc3}+P'_{sc3})\}\alpha[-\frac{3\hat{a}T_{sl}^2}{2}-\frac{3\hat{a}t_2^2}{2}-\frac{5\hat{b}T_{sl}^3}{6}-\frac{2\hat{b}t_2^3}{3}\quad\frac{5\hat{c}t_2^4}{12}+3\hat{a}T_{sl}t_2+$$
$$\hat{b}T_{sl}^2t_2+\frac{\hat{b}T_{sl}t_2^2}{2}-\frac{\hat{c}t_2^3T_{sl}}{3}-\frac{2\hat{c}T_{sl}^3t_2}{3}]+(\frac{n_i+1}{2n_i})P_i\mu L_t\frac{1}{2}\{\hat{p}_{c2}+\frac{1}{4}(\hat{p}_{c1}+\hat{p}'_{c1}+\hat{p}_{c3}+\hat{p}'_3)\}Q_{oq}+\frac{T_n}{T_{sl}}\quad[m_{tc_1}+(2\hat{D}V'_tm_{tc_2}+$$
$$\hat{D}\hat{m}_{tc_3}V'_tp_wQ_{oq})+((2\hat{D}c_{e1}+\hat{D}c_{e2}Q_{oq})(1-\xi'(1-e^{-\chi'\hat{G}_c}))-\frac{\hat{b}(t_2-t_0)}{2})]+G_{tc}T_{sl}+P_{dc}[-2\hat{a}t_0+\quad\frac{\hat{a}t_1^2\theta}{2}-C_Re^{-\beta t_0}+\frac{\hat{c}t_2^3}{3}-$$
$$\frac{\hat{c}(t_2^2-t_1^2)}{\beta}+\hat{b}t_2(t_2-t_0)-\hat{c}t_2^2(t_2-t_0)+\frac{2\hat{c}t_2(t_2-t_1)}{\beta}+(C_R-C_{ow})(\theta t_0-\frac{\theta^2t_0^2}{2})]-s_pE_i[(\hat{a}M_R+\frac{\hat{b}M_R^2}{2}-\frac{\hat{c}M_R^3}{3})(T_{sl}-t_2)+\frac{\hat{a}M_R^2}{2}+$$
$$\frac{\hat{b}M_R^3}{6}-\frac{\hat{c}M_R^2}{12}]+r_{ip}\frac{1}{2}\{\hat{p}_{c2}+\frac{1}{4}(\hat{p}_{c1}+\hat{p}'_{c1}+\hat{p}_{c3}+\hat{p}'_3)\}(-\frac{\hat{b}}{2\beta}-\frac{\hat{c}}{\beta^2})+\frac{\hat{c}}{3\beta}(t_2^3-M_R^3)+(t_2-M_R)(\frac{\hat{b}t_2}{\beta}-\frac{\hat{c}t_2^2}{\beta}+\frac{2\hat{c}t_2}{\beta^2})] \quad (28)$$

4. Numerical Example

The present section shows the applicability of both crisp and IF.

Note: In the case of all the models, some parameters are considered to be equal for the time factor and comparison purposes. Common parameter values for the models:

$\hat{C}_O=500, \hat{p}_c=5, H_{cw}=0.5, H_{cr}=0.6, C_{ow}=1500, \theta=0.04, n_i=4, P_i=0.02, L_t=0.4, \mu=0.5, T_n=2,$
$P_{sc}=5, \alpha=0.5, \beta=0.03, m_{tc_1}=0.1, m_{tc_1}=0.55, m_{tc_3}=1.4, c_{e1}=2.35, c_{e2}=1.3, \hat{D}=100, p_w=4, P_{dc}=0.1,$
$\xi=0.5, \chi=0.7, t_0=9, t_2=21, r_{ip}=0.15, s_p=40, E_i=0.12, \hat{a}=80, \hat{b}=0.7, \hat{c}=0.1, M_R=5.$

Parameters for the IF model:

$\tilde{\hat{p}}_c=(4,5,6;3,7), \hat{C}_o=(500,525,550;475,575)$ and $P_{sc}=(3,4,5;2,6)$.

4.1. Algorithm to Obtain the Optimum Result in the Numerical Approach

Step 1: Put all the known parameters in MATLAB.
Step 2: Put some value of T_{sl} and take $G_{tc}=1$.
Step 3: Consider Eqs. (25-28) to evaluate $T_{oc}/\tilde{T}_{oc}$.
Step 4: Use $G_{tc}=G_{tc}+1$ recursively up till the end of step 3, till getting the minimum $T_{oc}/\tilde{T}_{oc}$.
Step 5: Save the new G_{tc} and minimum$T_{oc}/\tilde{T}_{oc}$ obtained from step 4, and then go to step 6.
Step 6: Include $T_{sl}=T_{sl}+0.1$ andG_{tc}, obtained in step 5, and use Eqs. (25-28) to evaluate$T_{oc}/\tilde{T}_{oc}$. Repeat step 6 till the minimum. $T_{oc}/\tilde{T}_{oc}$is not found.
Step 7: Store the values of G_{tc}, $T_{sl}$and $T_{oc}/\tilde{T}_{oc}$.

Note: This algorithm is used both for crisp and IF models.

4.2. Optimum Results and Graphs

For both models, the optimum results for all the DVs and the total costs are presented in Table 3. The graphs (Figures 1 to 4) present the minimum total cost for two cases of the IF model concerning GT cost and total cycle length taken one by one in turn.

Table 3: Model architecture of the extra trees classifier

Type of Model	Case 1			Case 2		
	G_{tc}	T_{sl}	$\tilde{T}_{oc}$	G_{tc}	T_{sl}	$\tilde{T}_{oc}$
Crisp	10.8	62.5	9050.457	10.3	63	9844.005
IF	9.3	69.9	6408.070	5	70.7	7227.182

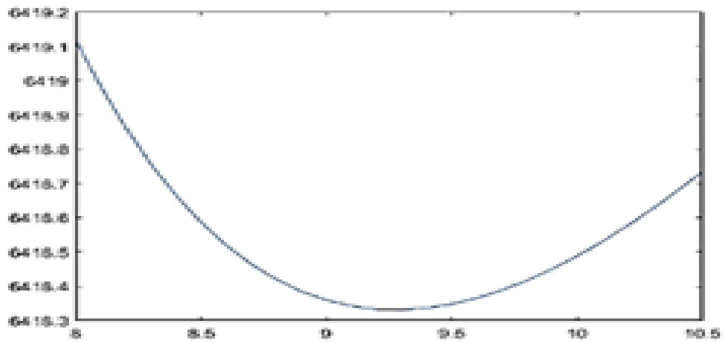

Figure 1: Convexity of for (for case 1)

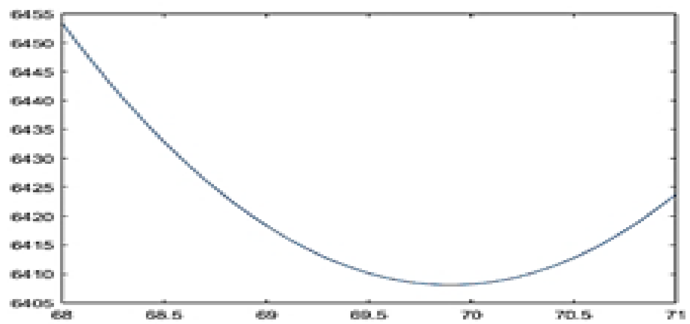

Figure 2: Convexity of for (for case 1)

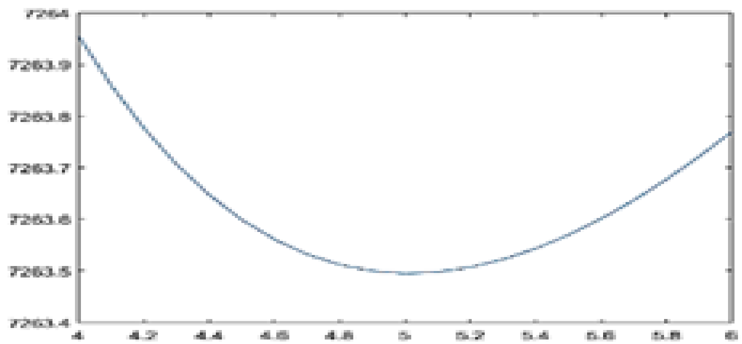

Figure 3: Convexity of for (for case 2)

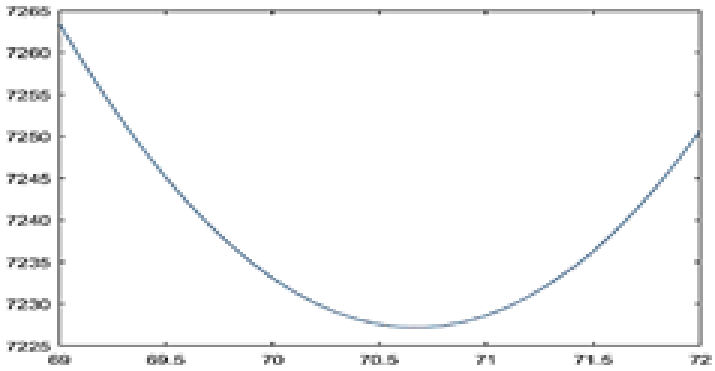

Figure 4: Convexity of for (for case 2)

4.3. Sensitivity Analysis

This section deals with analysing different results by changing the prime parameters available in both the EOQ models, which finally leads to valuable managerial insights. Here, we analyse the impact of changes in some of the parameters. $M_R, E_i, \hat{c}, \hat{b}, H_{cr}$ and H_{cw} in the range -10% to 10% of the optimal results of DVs of the model problems (Table 4):

Table 4: IF model

Case 1				Case 2		
$\mathbf{M_R}$	$\mathbf{G_{tc}}$	$\mathbf{T_{sl}}$	$\mathbf{\widetilde{T}_{oc}}$	$\mathbf{G_{tc}}$	$\mathbf{T_{sl}}$	$\mathbf{\widetilde{T}_{oc}}$
4.5	9.3	69.9	6214.236	5	70.9	7615.417
4.75	9.3	69.9	6301.478	5	70.8	7414.190
5.25	9.3	69.9	6507.451	5	70.6	6915.465
5.5	9.3	69.9	6612.214	5	70.5	6528.012
$\mathbf{E_i}$	$\mathbf{G_{tc}}$	$\mathbf{T_{sl}}$	$\mathbf{\widetilde{T}_{oc}}$	$\mathbf{G_{tc}}$	$\mathbf{T_{sl}}$	$\mathbf{\widetilde{T}_{oc}}$
0.108	9.3	69.8	6645.123	5	70.7	7401.963
0.114	9.3	62.8	6729.986	5	70.7	7332.753
0.126	9.3	70	6391.741	5	70.8	7145.159
0.132	9.3	70	6309.852	5	70.8	7074.745
$\mathbf{\hat{c}}$	$\mathbf{G_{tc}}$	$\mathbf{T_{sl}}$	$\mathbf{\widetilde{T}_{oc}}$	$\mathbf{G_{tc}}$	$\mathbf{T_{sl}}$	$\mathbf{\widetilde{T}_{oc}}$
0.09	10	72.5	6982.562	5.8	71.9	7999.150
0.095	9.7	70.7	6661.162	5.4	71.2	7730.521
0.105	9.2	69.1	6215.786	4.7	70	7445.105
0.11	8.9	67.2	6087.125	4.4	68.8	7659.483
$\mathbf{\hat{b}}$	$\mathbf{G_{tc}}$	$\mathbf{T_{sl}}$	$\mathbf{\widetilde{T}_{oc}}$	$\mathbf{G_{tc}}$	$\mathbf{T_{sl}}$	$\mathbf{\widetilde{T}_{oc}}$
0.63	9.9	69.8	6472.012	4.8	70.9	7389.569
0.665	9.9	69.9	6358.549	4.9	70.8	7415.018
0.735	9.6	70	6745.269	5.1	70.6	7523.569
0.77	9.6	70.1	7012.325	5.2	70.5	7898.235
$\mathbf{H_{cr}}$	$\mathbf{G_{tc}}$	$\mathbf{T_{sl}}$	$\mathbf{\widetilde{T}_{oc}}$	$\mathbf{G_{tc}}$	$\mathbf{T_{sl}}$	$\mathbf{\widetilde{T}_{oc}}$
0.54	9.3	69.9	6312.158	5	70.7	7199.791
0.57	9.3	69.9	6345.125	5	70.7	7201.095
0.63	9.3	69.9	6452.582	5	70.7	7284.705
0.66	9.3	69.9	6542.290	5	70.7	7285.010
$\mathbf{H_{cw}}$	$\mathbf{G_{tc}}$	$\mathbf{T_{sl}}$	$\mathbf{\widetilde{T}_{oc}}$	$\mathbf{G_{tc}}$	$\mathbf{T_{sl}}$	$\mathbf{\widetilde{T}_{oc}}$
0.45	9.3	69.9	6332.043	5	70.7	7109.369
0.475	9.3	69.9	6341.251	5	70.7	7169.589
0.525	9.3	69.9	6459.661	5	70.7	7254.874
0.55	9.3	69.9	6468.875	5	70.7	7312.298

4.4. Managerial Insights

- In the case of the increase in H_{cw}, there is a less significant change in total cost. Whereas an increase in H_{cr} leads to a very significant change in the total cost, which is almost the same percentage of change in H_{cr}. To make the model more cost-effective, the manager should try to keep H_{cr} under control to restrict the optimum total cost.
- It can be observed from Table 3 that when the permissible delay time M_R increases, the average total cost decreases. This suggests that the retailer's total costs drop after taking advantage of the credit period.
- Additionally, we see that the annual total cost falls as the rate of interest increases. E_i rises. This suggests that the retailer should prioritise a delay in payment option since the increase. E_i will result in higher earnings for the retailer.
- The optimal solutions of T_{sl} and $T_{oc}/\widetilde{T}_{oc}$ are sensitive to changes in the parameter $\hat{b}/\hat{c}$, that is T_{sl} and $T_{oc}/\widetilde{T}_{oc}$ decrease when$\hat{b}/\hat{c}$ increases. Thus, the value of $\hat{b}/\hat{c}$ The product may change depending on market demand.
- If $\hat{c}$ Increasing by 10% would not allow the optimum GT cost and the replenishment period to be reached. Therefore, the manager should have control over the percentage change in $\hat{c}$ to achieve the corresponding cost $\widetilde{T}_{oc}$.

Now we mention the advantages of IF models: i) Gives the minimum total cost. ii) It works efficiently, and the implementation part is elementary. Iii) Best to solve an inventory problem with uncertainty. iv) Through sensitivity analysis, suitable managerial insights are drawn.

5. Conclusion

This work investigates a two-warehouse sustainable inventory model with an STC, in which the suppliers usually allow a delay in the payment period for the retailers. Here, we have formulated a non-instantaneously deteriorating model that minimises the impact of the greenhouse effect due to transportation, while also minimising the GT cost, which significantly affects the total cost. The suggested model optimises G_{tc}, T_{sl}, and $T_{oc}/\widetilde{T}_{oc}$ in both the crisp and IF senses. Under the consideration of vagueness in the per unit purchasing cost, ordering cost, and shortage cost, we introduced an intuitionistic fuzzy cost, and also the $\widetilde{T}_{oc}$is defuzzified for the models by the $\alpha, \beta -$cut method. The optimality criteria are verified and validated for all the cases. The crisp and IF models give the optimum total cost to be 9050.457 and 6408.070, respectively, for case 1 and 9844.005 and 7227.182 for case 2. Again, G_{tc}, T_{sl} The data are presented in paired form for the crisp model: Case 1 (10.8, 62.5) and Case 2 (10.3, 63).

For the IF model, the data are as follows: Case 1 (9.3, 69.9) and Case 2 (5, 70.7). Here G_{tc}, T_{sl} and $T_{oc}/\widetilde{T}_{oc}$ are evaluated for the model problems, leading to a better result for the IF problem than the crisp case. In the present case, we have only considered the emissions due to transportation, but that may occur from holding the items in both RW and OW. Again, this research has not included fuzziness from other costs, such as holding cost, cyclic capital cost, and transportation costs, as well as the fuzziness in the demand function. This topic is highly challenging, as it has immense applications in different fields like health, business, sustainability, and climate. Further, this work can be extended by including multiple trade-credit options and considering other emissions sources. The work may thrive upon quality discounts and the demand, depending on both advertising and the price of items.

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Data Availability Statement: The study utilizes a dataset related to the effects of green technology investment and trade-credit facility in a two-warehouse fuzzy inventory model under partially backlogged shortages. The dataset can be made available upon reasonable request to the corresponding author.

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